Pulsating Mode of Flame Propagation in Two-Dimensional Channels

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Flame propagation in channels and cracks is a problem of considerable interest with applications in many practical combustion devices, in fire hazard scenarios, and in the emerging micropropulsion technologies. Understanding the dynamics and stability characteristics of flame propagation in channels is, therefore, important both for fundamental research as well as for practical applications. In this work, we examine the propagation of a premixed flame front in a two-dimensional channel in the presence of a Poiseuille flow. Our primary objective is to determine within the flammable regions the structure of the flame front and the conditions that result in steady propagation and those leading to a pulsating mode of propagation. Special attention is given to the difference between propagation in narrow and wide channels, heat losses to the channel's walls, and an imposed flow that either supports or opposes the propagation. In general, flame oscillations are found to occur in mixtures for which the effective Lewis number is sufficiently large. They are more likely to occur in narrow or wide channels and particularly at near-extinction conditions where the critical Lewis number is reduced to physically accessible values.

Nomenclature

a	=	half-channel's width
$\mathcal{B}$	=	pre-exponential factor of the reaction rate
$\mathcal{D}_{\text{th}}$	=	thermal diffusivity of the combustible mixture
d_q	=	quenching distance
E	=	activation energy
k	=	heat loss intensity
Le	=	Lewis number
Le_c	=	critical Lewis number for the onset of planar oscillations
l_{th}	=	laminar flame thickness, $= \mathcal{D}_{\text{th}}/S_L$
$\dot{m}$	=	total burning rate
R^0	=	universal gas constant
S_L	=	laminar flame speed
S_L^0	=	laminar flame speed for unity Lewis number
T	=	temperature
T_a	=	adiabatic flame temperature
t	=	time
U	=	flame propagation speed (relative to the walls)
u	=	axial velocity
u_0	=	maximum velocity in the channel (at the centerline)
$\bar{u}$	=	average velocity in the channel
$\langle u \rangle$	=	mean flow rate
x	=	axial coordinate
xy	=	transverse coordinate
Y	=	mass fraction of deficient reactant in the mixture
α	=	heat-release parameter, $= (T_a - T_u)/T_a$
β	=	Zeldovich number, $= E(T_a - T_u)/R^0 T_a^2$
ϵ	=	ratio of flame thickness to half-channel's width, $= l_{\text{th}}/a$
Θ	=	nondimensional temperature, $= (T - T_u)/(T_a - T_u)$
ω	=	reaction rate
$\langle \rangle$	=	time average

Subscripts

f	=	flame
u	=	fresh mixture

I. Introduction

IN recent studies[1−3] the structure of steadily propagating two-dimensional flame fronts in narrow, moderate, and wide channels has been examined, accounting for the effects of conductive heat loss to the channel's walls, an imposed convective field, and differential diffusion exhibited by nonunity Lewis numbers. Besides their fundamental importance in studies on flammability limits and flame-flow interaction, these problems have direct relevance to various applications. Flame spread through ducts and cracks can be a fire hazard. The gas flowing into crevice volumes of an internal combustion engine can support or prevent a flame from penetrating the narrow entrance, thus affecting the overall efficiency of the engine and possibly contributing to the emission of pollutants. In the development of miniaturized combustion devices, one is faced with issues concerning flame propagation in narrow gaps resulting from the reduction in scale.

Flammable regions of a combustible mixture are not only affected by flame propagation rates, but also by their stability properties. Spontaneous flame oscillation within operating conditions can hinder the efficiency of combustors so that fundamental knowledge regarding instability mechanisms would be extremely valuable in design considerations.

It is well known that the disparity between the thermal diffusivity of the mixture and the molecular diffusivities of the individual reactants is responsible for the spontaneous oscillation of a plane premixed flame. Spontaneous oscillations occur when the effective Lewis number of the mixture Le exceeds a critical value Le_c, usually larger than unity. The effective Lewis number[4] is defined as a weighted average of the separate Lewis numbers associated with the fuel and oxidizer, with that corresponding to the deficient component weighting more heavily; in lean mixtures it corresponds to the fuel Lewis number and in rich mixtures to the oxidizer Lewis number. For a planar flame, asymptotic theories based on a large Zeldovich number β predict[5] a critical value $Le^* = 1 + 4(1 + \sqrt{3})\beta^{-1}$. It has been later verified numerically[6] that the critical Lewis Le_c

tends to Le^* as β becomes large. The numerical calculations, however, show that Le_c is quite sensitive to the specified value of β; with $\beta = 10$ it was found that $Le_c \approx 3.58$, which is nearly 70% larger than $Le^* = 2.09$. In any case, the predicted values of the Lewis number beyond which oscillations occur are quite large and thus not easily accessible in ordinary combustible mixtures. For this reason experimental observations of pulsating planar flames are rather scarce and occur only in burner-stabilized flames,[7] where the influence of conductive losses to the burner rim act to lower the critical Lewis number considerably.[8−10]

When a flame propagates in a channel, heat losses from the flame occur naturally by conduction to the walls, and it is thus possible that the critical Lewis number for the onset of oscillations is reduced to physically realistic values. On the other hand, the flame in a channel is generally curved, and curvature has a stabilizing influence. Because of their relevance in applications, it is of fundamental and practical interest to investigate the conditions that result in steady propagation and those that lead to a pulsating mode of propagation. Previous works on flame oscillation in premixed systems were primarily concerned with planar flames, and planar stretched flames, using one-step chemistry and constant properties[5,6,8−11] or detailed chemistry and transport.[12−15] There were also studies on the onset of oscillations in weakly nonplanar flames that resulted as bifurcated solutions of an originally planar flame.[16] The present work is different in that the basic state is a nominally two-dimensional front. Other related studies of flames in channels were concerned with flow instability, for example, tulip-flame formation and flame acceleration caused by confinement.[17−19]

In this work we examine the possible development of a pulsating mode of flame propagation in channels accounting for the effects of conductive losses to the walls and an imposed flow that either supports or opposes the propagation. The channel's width is treated as a parameter, and the analysis spans the whole range from narrow to wide channels. For simplicity the chemistry is modeled by a one-step overall reaction but with finite rate chemistry and a relatively large Zeldovich number. To suppress hydrodynamic disturbances, the analysis is carried out in the context of a diffusive-thermal model. The mathematical formulation is presented next, followed by a description of the numerical approach and a discussion of the results and their physical significance.

II. Formulation

We consider a two-dimensional deflagration wave that travels in an infinitely long channel of width $2a$, separating the fresh cold mixture on the left end of the channel from the hot combustion products (Fig. 1). For simplicity, we assume that the mixture of temperature T_u comprises a deficient combustible component that determines the rate of progress of the chemical reaction, with mass fraction Y_u. A diffusive-thermal, or constant-density model, which allows examining the propagation of a flame in a prescribed flow while neglecting the effect that the flame has on the flow field, is considered. The flow here is assumed to be a Poiseuille flow with the velocity taken to be positive when directed from the unburned toward the burned gas and negative when directed from the burned toward the unburned gas (Fig. 1). The chemical activity is described by a one-step overall reaction and proceeds at a rate given by an Arrhenius law with activation energy E and a preexponential factor $\mathcal{B}$.
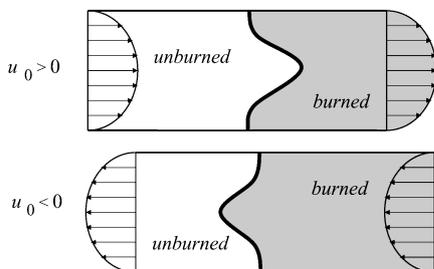

Fig. 1 Schematic of flame propagation in a channel opposed/supported by a flow.

We select as a unit length the half-width of the channel a, as a unit speed the laminar flame speed S_L, and as a unit time a/S_L. The axial velocity is given by $u = u_0(1 - y^2)$, where u_0 is the centerline velocity in units of S_L. The ratio of the laminar flame thickness l_{th} to a is denoted by $\epsilon \equiv l_{th}/a$ and treated as a parameter; here $l_{th} = \mathcal{D}_{th}/S_L$ with $\mathcal{D}_{th}$ the thermal diffusivity of the mixture. The mass fraction of the deficient reactant is normalized by its value in the fresh mixture Y_u, and a nondimensional temperature $\Theta = (T - T_u)/(T_a - T_u)$ is defined with T_a the adiabatic flame temperature. The dimensionless governing equations, written in a frame attached to the propagating flame, are

$$\frac{\partial \Theta}{\partial t} + \left[U + u_0(1 - y^2) \right] \frac{\partial \Theta}{\partial x} - \epsilon \nabla^2 \Theta = \omega \qquad (1)$$

$$\frac{\partial Y}{\partial t} + \left[U + u_0(1 - y^2) \right] \frac{\partial Y}{\partial x} - \epsilon Le^{-1} \nabla^2 Y = -\omega \qquad (2)$$

with the reaction rate given by

$$\omega = \frac{1}{2\epsilon Le} \beta^2 Y \exp\left\{ \frac{\beta(\Theta - 1)}{1 + \alpha(\Theta - 1)} \right\} \qquad (3)$$

Here Le is the Lewis number based on the mass diffusivity of the deficient reactant in the mixture, $\beta = E(T_a - T_u)/R^0 T_a^2$ is the Zeldovich number, and $\alpha = (T_a - T_u)/T_a$ is the heat-release parameter. In the governing equations $U(t)$ is the propagation speed, measured with respect to the walls, and taken to be positive when the flame is propagating toward the unburned gas. It is an eigenvalue in this problem that remains to be determined as part of the solution. Finally, we note that in writing Eq. (3) we have used the large activation-energy-asymptotic expression

$$S_L = \sqrt{\frac{2\mathcal{D}_{th} Le \mathcal{B}}{\beta^2}} \exp\left(\frac{-E}{2R^0 T_a} \right) \qquad (4)$$

for the laminar flame speed.

Far to the left, the chemistry is frozen because of a sufficiently large β, so that $Y = 1$ and $\Theta = 0$ as $x \to -\infty$. Far to the right all properties are assumed uniform, and $\partial Y/\partial x = \partial \Theta/\partial x = 0$ as $x \to \infty$. The symmetry condition along the axis of the channel implies that

$$\frac{\partial Y}{\partial y} = \frac{\partial \Theta}{\partial y} = 0 \qquad \text{at} \qquad y = 0 \qquad (5)$$

and the conditions corresponding to impermeable and nonadiabatic walls are

$$\frac{\partial Y}{\partial y} = 0, \qquad \frac{\partial \Theta}{\partial y} = -\frac{k\Theta}{\beta} \qquad \text{at} \qquad y = 1 \qquad (6)$$

where k/β measures the intensity of heat loss by conduction (the insertion of the scaling factor β has been made simply for convenience). Hence $k = 0$ corresponds to adiabatic walls and $k = \infty$ to cold walls held at the same temperature as that of the fresh gases. The symmetry condition (5) precludes the development of time-dependent asymmetric solutions, an issue that will be addressed in a future study.

The total burning rate, representing the mass of (deficient) reactant consumed per unit time relative to that consumed by a planar flame propagating in the same channel, is given by

$$\dot{m} = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-1}^{1} \omega \, dx \, dy$$

For steady burning, integrating Eq. (2) across the channel and using boundary conditions (5) and (6) yields $\dot{m} = U + \bar{u}$, where $\bar{u} = 2u_0/3$ is the mean flow rate through the channel. Thus, the burning rate is equal to the propagation speed of the flame relative to the mean flow rate. For time-periodic solutions, a similar relation exists between the average burning rate during a cycle and the average propagation speed.

III. Numerical Procedure

The unsteady problem consisting of Eqs. (1) and (2) with the stated boundary conditions has been solved numerically. The computations are carried out in a finite domain $-L < x < L$ on a rectangular grid. Typically 40 grid points were used in the y direction, distributed uniformly in $(0, 1)$. In the x direction 200–400 grid points were used nonuniformly distributed according to

$$x_i = \left[\hat{x}_i / \left(2 - \hat{x}_i^2 \right)^c \right] L \tag{7}$$

where the points $\hat{x}_i$ were spread uniformly in $(-1, 1)$ and the constant c was assigned the value 4 for calculations in wide channels and reduced to 2 for calculations in narrow channels. Note that formula (7) ensures that more grid points are allocated in the interval $-0.2 < x < 0.2$ that always includes the reaction zone located near $x = 0$. The initial conditions are chosen such that the reaction zone is at $x = 0$. A marching procedure is then used to advance the solution in time. The propagation speed U at each time step is determined by holding the location $(x_r, 0)$, where $Y = Y_r$ fixed. The reference value Y_r was chosen sufficiently small so that x_r well approximates the location of the reaction zone along the axis. The reaction zone therefore remains near the origin at all times, and the denser distribution of grid points in that region allows for an appropriate resolution of its structure. Spatial derivatives were approximated using a second-order central difference scheme or the sixth-order compact scheme of Lele[20] depending on the special need. The marching procedure was based in most cases on a first-order discretization in time; often, however, a fourth-order discretization (5-4 scheme of Carpenter and Kennedy[21]) was used to verify the accuracy of the solution. Because more grid points are allocated to the neighborhood of the reaction zone, the time step is restricted by the diffusion terms in the governing equations. The presence of the highly nonlinear reaction-rate term, therefore, requires indirectly choosing the time step to be sufficiently small so as to ensure numerical stability. Consequently the higher-order methods are very time consuming and used in selected cases to test the accuracy and consistency of the results. A typical time step for the lower order schemes was $\Delta t = 10^{-4}$. Typically the initial conditions chosen to determine the evolution in time were based on a planar flame front. A continuation method was adopted when calculating the solution for consecutive values of a given parameter. The independence of the results to the number of grid points and to the length of the domain was tested repeatedly by refining the grid and/or extending L appropriately.

The results presented in this paper use the realistic values $\beta = 10$ for the Zeldovich number and $\alpha = 0.8$ for the heat-release parameter. Although for a finite value of β the reaction term does not vanish identically no matter how small Θ is (the cold-boundary difficulty), its magnitude becomes exponentially small in just a short distance away from the reaction zone and hence is practically zero further upstream. For example, for a planar flame the temperature drops to $\Theta = 0.05$ within a distance $x = 0.5$, which constitutes less than 4% of the preheat zone, and the reaction rate is then below 10^{-17}.

For a finite value of β, the laminar flame speed S_L differs from the asymptotic value (4) used in the nondimensionalization. It can be either calculated independently using a one-dimensional code or obtained from the present calculations as the propagation speed of a flame in a wide channel under adiabatic conditions ($k = 0$) and no imposed flow ($u_0 = 0$). For unity Lewis number, $\beta = 10$, and with $\epsilon = 0.1$, we found that $U = 0.918$ in agreement with earlier results[6] obtained from calculations based on a one-dimensional formulation. The proper reference speed for unity Lewis number flames S_L^0 is therefore obtained by multiplying the asymptotic value by 0.918. Rather than repeating this calculation for a whole range of Lewis numbers, we have adopted the asymptotic dependence $S_L \sim Le^{1/2}$, which was verified to be sufficiently good for our present purpose. The laminar flame speed is thus taken to be $S_L = Le^{1/2} S_L^0$, and velocities reported next have been renormalized based on this value. Finally, we note that because a time-dependent procedure is being used, the long-time behavior of the solution obtained in the calculations is necessarily a stable state.

IV. Steady Propagation

We start by presenting solutions that correspond to steady propagation, illustrating the effects of a convective flow, heat losses, and differential diffusion.

Figure 2 shows representative results for unity Lewis number that illustrate the effects of an imposed flow on adiabatic flames in a relatively wide channel ($\epsilon = 0.1$). The curves in these figures correspond to reaction-rate contours that span the same range of ω starting at some sufficiently small value. In each case the propagation speed U is indicated in the figure. In the absence of a flow ($u_0 = 0$), the flame is planar and propagates to the left at a speed $U = 1$. In the presence of a convective flow, the flame is curved as illustrated in Fig. 1, and, as a result of the larger flame surface area, there is an increase in burning rate $\dot{m}$. For small $u_0 > 0$ the flame propagates to the left against the flow, a condition referred to as flashback. Being opposed by the flow, the propagation speed is smaller than the speed of a planar flame. When u_0 reaches a critical value u_0^{cr}, the flame is stabilized by the flow, that is, the propagation speed $U = 0$. For the case presented in the figure, $u_0^{cr} \approx 3.4$, implying that for stabilization the mean flow rate $\langle u \rangle$ needs to be approximately twice the laminar flame speed. For $u_0 > u_0^{cr}$ the flame is blown off by the flow, propagating to the right at a speed that increases with increasing u_0. The asymptotic approximation[1]

$$U \sim 1 - 2.04 \left(u_0 \epsilon \right)^{\frac{2}{3}} \qquad \text{for} \qquad u_0 > 0$$

derived for small ϵ yields sufficiently accurate results for U; in particular the critical value u_0^{cr} for $\epsilon = 0.1$ is estimated to be ≈ 3.4, very near the computed value. When the flow is directed from the burned toward the unburned gas, the flame is always blown off by the flow and propagates to the left. The asymptotic approximation for small ϵ, in this case, takes the form

$$U \sim 1 + |u_0| - (2|u_0|)^{\frac{1}{2}} \epsilon \qquad \text{for} \qquad u_0 < 0$$

Figure 3 illustrates the effects of heat loss, retaining the Lewis number equal to one and considering, as before, a relatively wide channel ($\epsilon = 0.1$). The moderate value of $k = 7$ was chosen in these calculations. To enable comparison with the adiabatic case, the same range of ω as in Fig. 2 was chosen for the reaction-rate contours. We see that even without a flow the flame is now curved; it is convex toward the unburned gas when $u_0 = 0$ and propagates at the speed $U = 0.95$, which is approximately 5% below the laminar flame speed. The reduction in speed is clearly caused by a decrease in burning rate caused by heat losses. The intensity of the reaction rate varies from a maximum ≈ 1.7 to the lowest value ≈ 0.47, with a
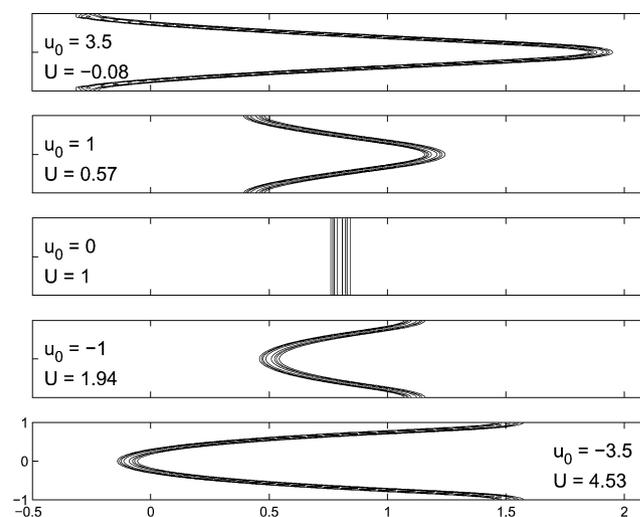
Fig. 2   Reaction-rate contours for flame propagation in an adiabatic channel under various flow conditions. Calculated with $k = 0$, $Le = 1$, and $\epsilon = 0.1$.

dead space developing near the walls and widening as k increases. In the absence of a flow, the flame in such a wide channel always survives near the center, no matter how large k is. For $u_0 = 1$ the reaction intensity is reduced significantly, varying from a maximum of ≈ 1.41 to a value of ≈ 0.09 near the walls; the dead space is clearly seen in this case in the figure. Unlike the adiabatic case, the propagation speed $U = -0.055$ is now negative, and the flame is blown off by the flow. The critical velocity u_0^{cr} for flame stabilization decreases with increasing k, implying that it is easier to stabilize a flame in the presence of heat losses as is well known from experience with burners. By further increasing the flow velocity (not shown in the figure), total flame extinguishment results; in the present conditions this would occur when $u_0 \approx 1.2$. On the other hand, for $u_0 < 0$ the flame appears to span the whole channel's width. Heat losses to the walls have a minimal effect on the propagation speed of the flame, which, supported by the flow, is being blown off sufficiently fast through the channel. The flame is therefore much more sensitive to heat losses when it propagates against the flow than when it propagates with the flow.

Next, we consider the effects of differential diffusion of heat and mass. The focus here is on $Le > 1$, and variations in Le can be realized by changing, for example, the mass diffusivity of the deficient reactant in the mixture by dilution. In the absence of an imposed flow, a steadily propagating flame under adiabatic conditions remains planar and propagates at a speed $U = S_L^0 Le^{1/2}$. The propagation speed thus increases with increasing Lewis number. In
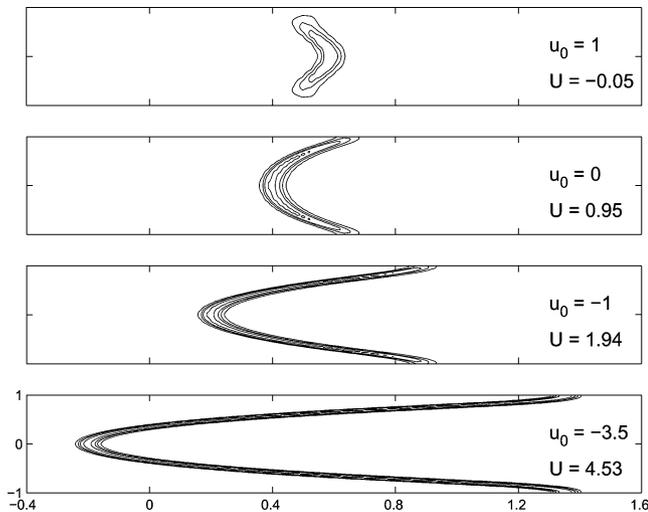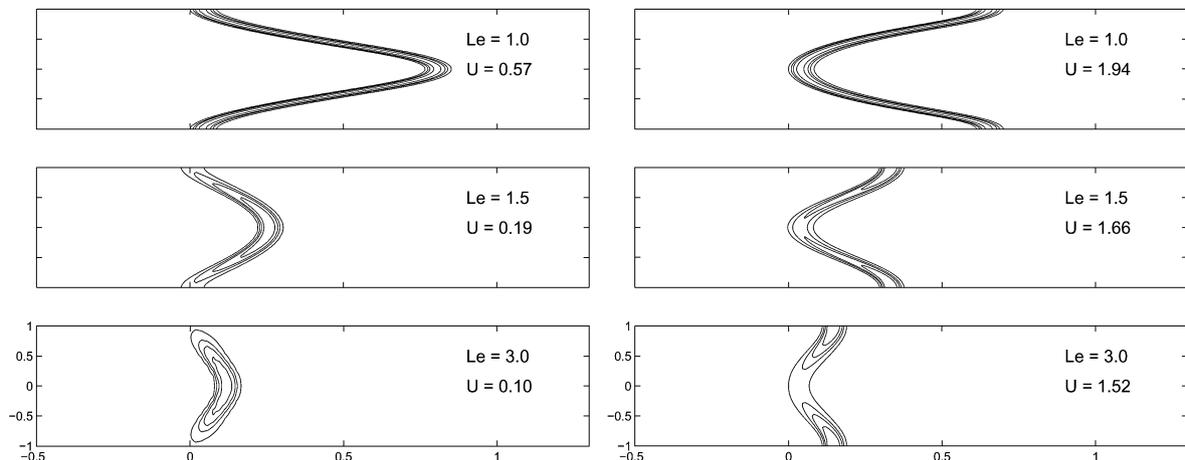
the presence of a flow, the flame is curved, as already shown in Fig. 2, and the increase in surface area leads to an increase in overall burning rate. When the Lewis number is larger than one, thermal diffusivity tends to spread out temperature differences at a rate that exceeds the rate of diffusion of the reactants toward the flame. As a result, the flame flattens out irrespective of the flow direction. This is illustrated in Fig. 4 for flames propagating in a wide channel ($\epsilon = 0.1$) either opposed or supported by the flow ($u_0 = \pm 1$), under adiabatic conditions. For $u_0 = 1$ the flame, in all of the three cases shown in the figure, propagates to the left because the velocity $u_0 < u_0^{cr}$. The flame curvature decreases significantly with increasing Le with the burning intensified near the center of the channel and weakened near the walls. The overall effect is a reduction in propagation speed or total burning rate. For example, when $Le = 1$ the reaction rate is nearly uniform along the flame with $\omega_{max} \approx 1.7$, but when $Le = 3$ the reaction rate $\omega_{max} \approx 2.1$ at the center of the channel and $\omega_{max} \approx 0.3$ near the walls. The propagation speed decreases from $U = 0.57$ when $Le = 1$ to a value as low as $U = 0.1$ when $Le = 3$. For $u_0 = -1$ the flame curvature also decreases with increasing Le, but in this case the burning is intensified near the walls and weakened near the center of the channel. When $Le = 1$, for example, the burning rate along the flame is nearly uniform with $\omega_{max} \approx 1.7$, but when $Le = 3$ the reaction rate $\omega_{max} \approx 3.0$ near the walls and $\omega_{max} \approx 0.4$ at the center of the channel. The propagation speed again decreases, from $U = 1.94$ when $Le = 1$ to $U = 1.52$ when $Le = 3$. By further increasing Le, the burning becomes more and more localized, either near the center or near the walls. Thus, the disparity between the thermal diffusivity of the mixture and the mass diffusivity of the limiting reactant would lead to local flame extinction even under adiabatic conditions. For a Lewis number sufficiently larger than one, local extinction occurs near the leading edge of the flame, that is, near the walls when $u_0 > 0$ and near the centerline when $u_0 < 0$, in contrast to the local extinction that would result when the Lewis number is sufficiently smaller than one, which occurs then at the trailing edge of the flame.[3] For thin flames ($\epsilon \ll 1$) these behaviors are consistent with the predictions based on the well-known flame speed—flame stretch relation, as discussed in detail in the referenced paper. In narrow channels, when the flame is relatively thick ($\epsilon \gg 1$) and transverse diffusion is equally important as axial diffusion, differential diffusion is not accentuated, and the burning rate is independent of Lewis number.

V.   Unsteady Propagation

As pointed out in the Introduction, it is well known that a planar flame becomes unstable when the Lewis number exceeds a critical value Le_c, giving rise to spontaneous oscillations during its propagation. Motivated by this result we examine, in the following, the long-time behavior of the two-dimensional flame structures that

Fig. 3   Reaction-rate contours for flame propagation in a channel under various flow conditions, accounting for conductive heat loss to the walls. Calculated with $k = 7$, $Le = 1$, and $\epsilon = 0.1$.

Fig. 4   Reaction-rate contours for flame propagation in an adiabatic channel, illustrating the effects of the Lewis number: the graphs on the left correspond to flames opposed by the flow ($u_0 = 1$) and those on the right to flames supported by the flow ($u_0 = -1$). Calculated for $k = 0$ and $\epsilon = 0.1$.

evolve as Le increases, starting with conditions that correspond to a steady propagating planar flame.
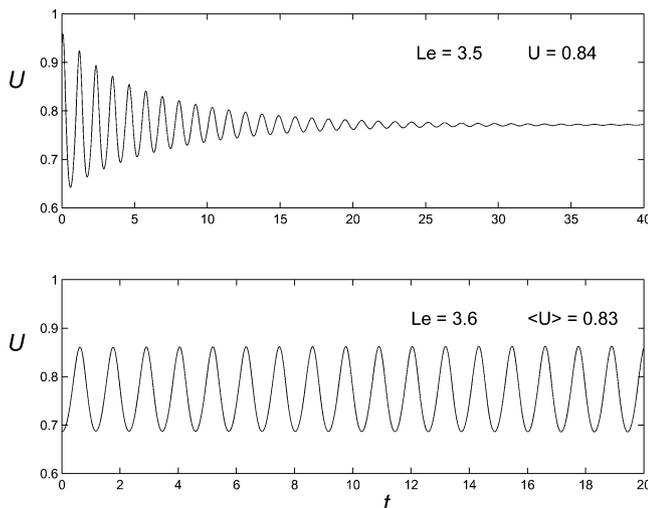
A. Adiabatic Flames

We consider first an adiabatic channel with $\epsilon = 0.1$ and no imposed flow. This case corresponds to a sufficiently wide channel, so that when steadily propagating the flame is planar. Spontaneous oscillations develop, as expected, when Le exceeds the critical value Le_c. The graph in Fig. 5 shows the dependence of U on time for two values of Le close to Le_c, here estimated to be ≈ 3.58. When $Le = 3.5$, the propagation speed tends to a constant, $U = 0.84$, as $t \to \infty$. In contrast, when $Le = 3.6$ a limit cycle develops for large t, corresponding to a pulsating propagation mode with an average speed $\langle U \rangle = 0.83$. The average speed was calculated from the relation

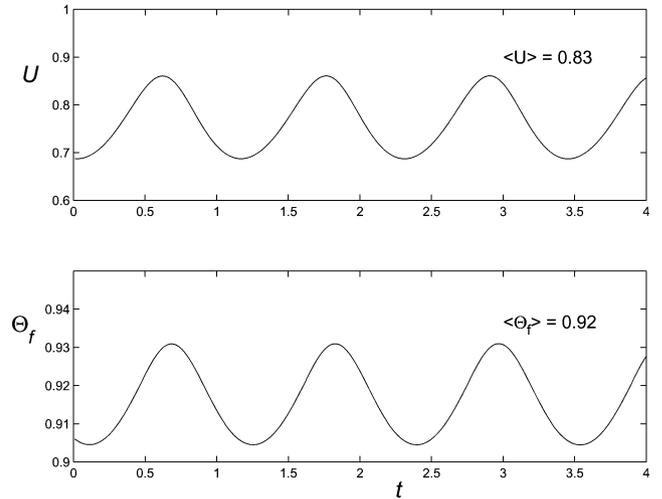$$\langle U \rangle = \frac{1}{\Delta t} \int_t^{t+\Delta t} U \, dt \tag{8}$$

with the interval Δt taken much larger than the period of the oscillation. Although the critical Lewis number Le_c can be determined more accurately by refining the calculations in the range (3.5, 3.6), this refinement is of no particular interest here and will not be pursued further. Note that Le_c is, within the accuracy of the calculations, equal to the value obtained for planar unbounded flames[6] based on one-dimensional calculations. Further calculations reveal that this result remains independent of ϵ, so that for adiabatic flames with no imposed flow the critical value where planar oscillations occur is well approximated by $Le_c \approx 3.58$.

The nature of the oscillations is shown in Fig. 6, where the propagation speed U and the flame temperature Θ_f are both plotted as a function of time. (The former is an enlargement of Fig. 5.) The flame temperature Θ_f is defined as the temperature at the location along the centerline of the channel, where the maximum reaction rate occurs, and the average flame temperature $\langle \Theta_f \rangle$ is calculated in a similar way as the average speed. We note that both Θ_f and U exhibit oscillations with the same frequency, and the maximum/minimum speed is obtained when the maximum/minimum temperature is reached.

By further increasing the Lewis number, the oscillations are sustained, as shown in Fig. 7, but the nature of the oscillations changes. The long-time behavior of the propagation speed $U(t)$ is plotted as a function of time, for several values of $Le > Le_c$. In each graph, the mean propagation speed $\langle U \rangle$ is indicated. Near the bifurcation point $Le_c \approx 3.58$, the oscillations have a sinusoidal behavior. For larger values of Le, the frequency of oscillations remains nearly constant, but their amplitude increases significantly. Furthermore, for large
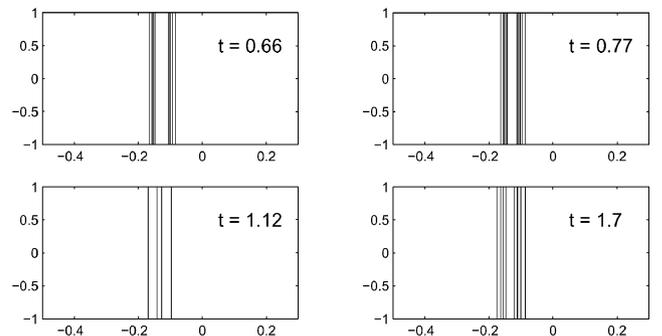

Fig. 6   Propagation speed U and flame temperature Θ_f as functions of time for the limit cycle obtained with $Le = 3.6$. Calculated for $k = 0$, $u_0 = 0$, and $\epsilon = 0.1$.
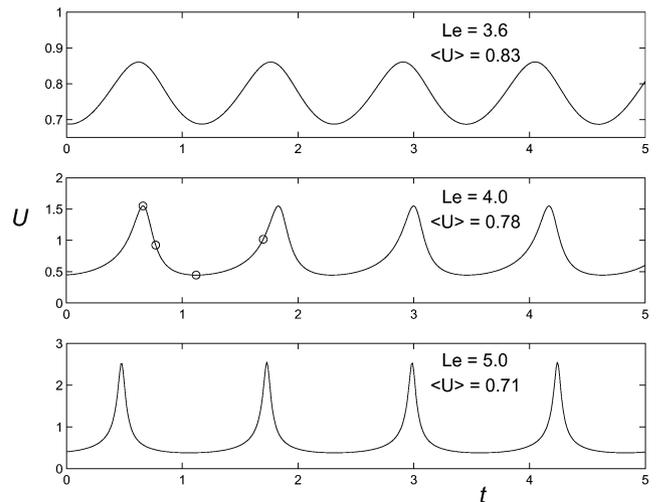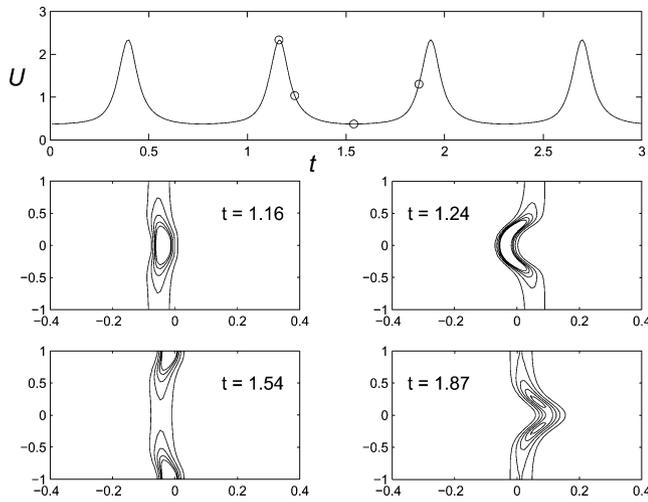

Fig. 7   Nature of flame oscillation for different increasing values of Le; calculated for $k = 0$, $u_0 = 0$, and $\epsilon = 0.1$.

Fig. 8   Reaction-rate contours for a planar adiabatic pulsating flame at four instances (marked in the preceding figure for the case $Le = 4$) within a cycle.

values of Le the flame performs relaxation oscillation characterized by intermittent long and short intervals within a period. The structure of the pulsating planar flame during a cycle is shown in Fig. 8, where reaction-rate contours for $Le = 4$ are plotted at four selected times marked in the graph displaying $U(t)$. Five contours spanning the range of $\omega = 0.3 - 1.5$ describe the flame at time $t = 0.66$, when the propagation speed reaches its maximum $U = 1.5$. Of the five contours only the lowest two, corresponding to $\omega = 0.3, 0.6$,
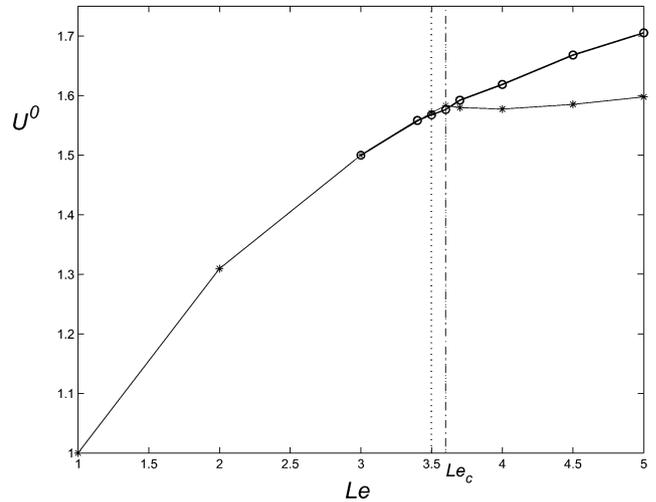

Fig. 5   Propagation speed U as function of time for two values of Le illustrating the approach to a steady state for $Le = 3.5$ and to a limit cycle for $Le = 3.6$. Calculated for $k = 0$, $u_0 = 0$, and $\epsilon = 0.1$.

Fig. 9 Reaction-rate contours of a nonplanar adiabatic pulsating flame at four instances within a cycle. Calculated for $u_0 = 0$, $k = 0$, $\epsilon = 0.1$, and $Le = 4$.

Fig. 10 Dependence of the propagation speed or mean propagation speed on the Lewis number Le; here U^0 is the propagation speed scaled with respect to the speed of an adiabatic planar flame with unity Lewis number. The critical Lewis Le_c for the onset of a planar or two-dimensional pulsating solution is identified by a vertical dashed–dotted line and dotted line, respectively, and the corresponding bifurcated solution branches are identified with the symbols $*$ or $\circ$.

describe the flame at time $t = 1.12$, that is, when it reaches its lowest speed $U = 0.5$. Hence, during the long time intervals within the cycle the reaction rate is weak, the temperature is relatively low, and the flame travels at a speed much lower than the laminar flame speed. In the short time intervals the reaction rate, and consequently the flame temperature, are sufficiently high to cause a rapid forward motion with a speed that significantly exceeds the laminar flame speed.

Planar oscillations beyond $Le_c \approx 3.58$ were obtained by choosing a planar unperturbed flame as initial condition, thus forcing the flame to retain its planar shape. The results differ in details if two-dimensional disturbances are permitted. The critical value Le_c is lowered slightly to ≈ 3.5, so that above this value the plane flame is already unstable, giving rise to an oscillating two-dimensional structure. Reaction-rate contours for $Le = 4$ spanning the range of $\omega = 0.3 - 1.5$ are shown in Fig. 9 at four instances within one period of oscillation; the representative times are marked in the graph displaying $U(t)$, which shows that the flame performs relaxation oscillation. During the long time interval within a cycle, the flame is nearly planar; the chemical reaction appears more intense near the walls and weaker along the centerline of the channel. During the short intervals separating the relaxation modes, chemical reaction is more intense near the center of the channel, and the flame is concave/convex toward the unburned gas intermittently. Finally, we note that nonplanar oscillations for $Le > Le_c$ were only found in wide channels or ϵ sufficiently small. These results are in full agreement with the linear stability results of planar flames,[5,16] which show that the most dangerous mode at the onset of instability is not the planar mode, but rather one with a finite wavelength, and it occurs at a Lewis number $Le = 1 + 32/3\beta$, which is less than Le^*. Clearly this mode becomes irrelevant in narrow channels, of width smaller than the critical wavelength.

We note that the trend that can be deduced from Fig. 7, suggesting that the mean speed $\langle U \rangle$ decreases with increasing Le, does not exhibit the correct dependence of the propagation speed on Lewis number. The reason is that velocities have been scaled with respect to S_L, which itself depends on Le. In Fig. 10 we have plotted in the ordinate the propagation speed scaled relative to the speed of an adiabatic planar flame with unity Lewis number S_L^0. Using the approximate asymptotic dependence $S_L \sim Le^{1/2} S_L^0$ for rescaling the ordinate simply amounts to multiplying U by $Le^{1/2}$. Selective calculations carried out by numerically solving the governing equations after rescaling velocities with respect to S_L^0, and similarly expressing the characteristic length l_{th} in terms of S_L^0 led to identical results. The graph thus shows the true dependence of the propagation speed or mean propagation speed on the Lewis number. The propagation speed of a steadily propagating planar flame increases with increasing Lewis number as predicted by the asymptotic formula (4), that

is, with a dependence proportional to $Le^{1/2}$. There is a small decrease in the mean propagation speed of the planar pulsating flame beyond the bifurcation point ($Le_c \approx 3.58$, or dash-dotted line), in agreement with the weakly nonlinear asymptotic result.[22] By further increasing Le, however, the mean propagation speed starts increasing again but at a very slow rate. This solution, however, is unstable to two-dimensional disturbances as discussed earlier. The mean propagation speed of the curved pulsating flame increases monotonically with Le, even beyond the bifurcation point, at a rate approximately proportional to $Le^{1/2}$. Note that the bifurcation point in this case (the dotted line) is slightly lower than 3.58, as noted earlier.

B. Heat Loss

Next we examine the effect of conductive heat losses to the channel's walls on the onset of oscillations. In assessing the effects of heat loss, it was found previously[2] that depending on the channel's width there are two possible modes of flame extinction: partial extinction near the walls in relatively wide channels and total extinction in narrow channels. The quenching distance d_q, defined as the width of the narrowest channel through which a flame can propagate in a stationary gas mixture, is associated with total extinction. The dependence of d_q on Lewis number was previously examined[23] for isothermal cold walls ($k = \infty$), where it was found that d_q increases with increasing Le. The onset of flame instability was not discussed in these studies. The present results based on time-dependent calculations show a similar trend; for any fixed value of the heat-loss intensity k, the Lewis number above which total extinction occurs decreases with increasing ϵ, namely, as the channel becomes narrower. Extinction curves that separate regions where flame propagation is, or is not possible, are shown in Fig. 11 by dashed curves for two values of k. To the left of the curve, flame propagation, whether steady or unsteady, is possible; to the right of the curve, propagation is not possible in any form. For given k, the solid curve identifies the onset of oscillations that occur when Le exceeds the critical threshold Le_c. The oscillatory states in the $Le - \epsilon$ parameter plane correspond to the region left of the dashed curves and above and to the right of the solid curves. Thus, for low values of $Le > 1$, extinction occurs where steady propagation fails. But for larger values of the Lewis number, the flame becomes unstable giving rise to a pulsating mode of propagation and extinction occurs when this new mode of propagation fails. Unlike the case of adiabatic walls, Le_c depends here on the channel's width, as shown in the graph by the solid curves. Oscillations are more likely to occur in narrow channels and, in particular, at near-extinction conditions
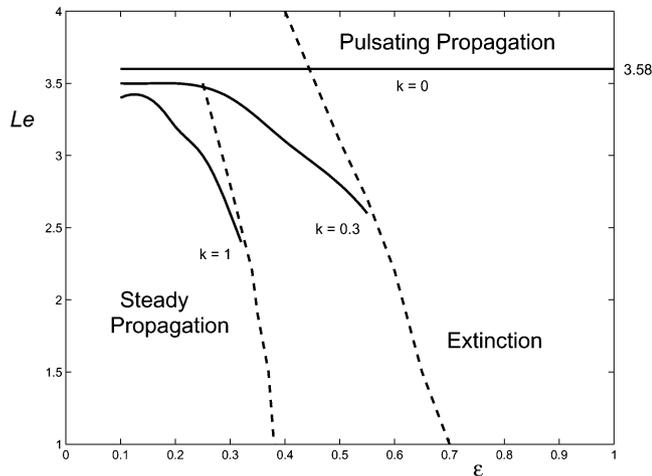
Fig. 11   Regions in the parameter plane, Lewis number Le or ϵ (inversely proportional to channel's width), where steady vs pulsating propagation occur in the absence of a flow ($u_0 = 0$). Illustrated for adiabatic walls ($k = 0$) and two values of the heat-loss coefficient k; ----, extinction limits.
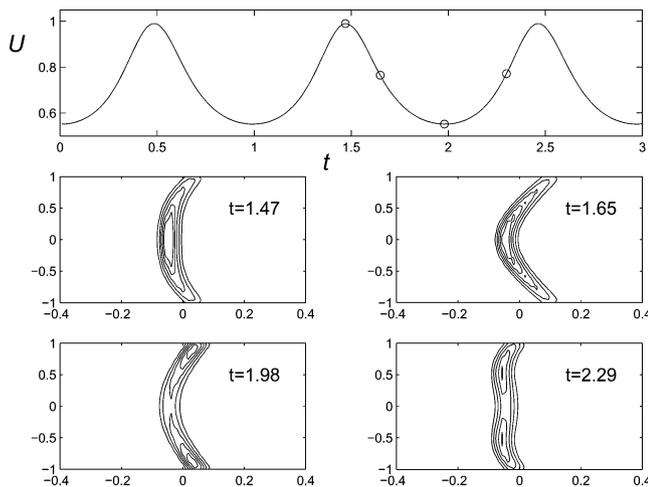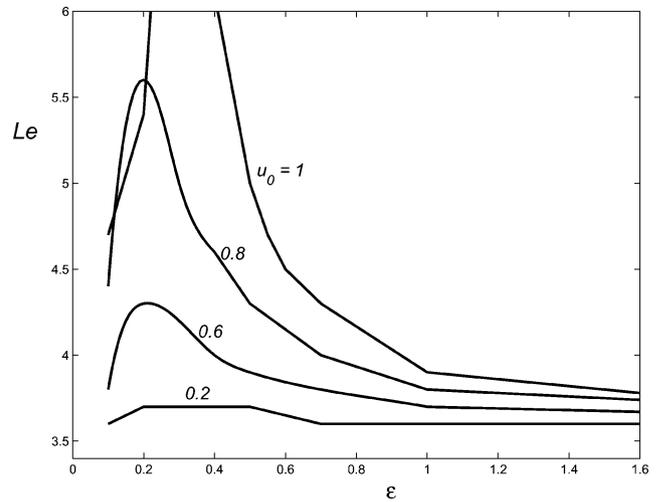

Fig. 12   Reaction-rate contours of a nonadiabatic pulsating flame at four different instances within a cycle, calculated for $u_0 = 0$, $k = 5$, $\epsilon = 0.1$, and $Le = 3$.

Fig. 13   Regions in the Le vs ϵ parameter plane where steady or pulsating propagation occur, in the presence of an opposed flow under adiabatic conditions.

Fig. 14   Regions in the Le vs ϵ parameter plane where steady or pulsating propagation occur, in the presence of an opposed flow under nonadiabatic conditions; calculated for $u_0 = 1$.

where Le_c has been reduced significantly. The critical Lewis number Le_c for the onset of oscillations is reduced from $Le \approx 3.58$ for the adiabatic case down to the more physically accessible values of $Le \approx 2.5$ for $k = 0.3$ and $Le \approx 2.2$ for $k = 1$.
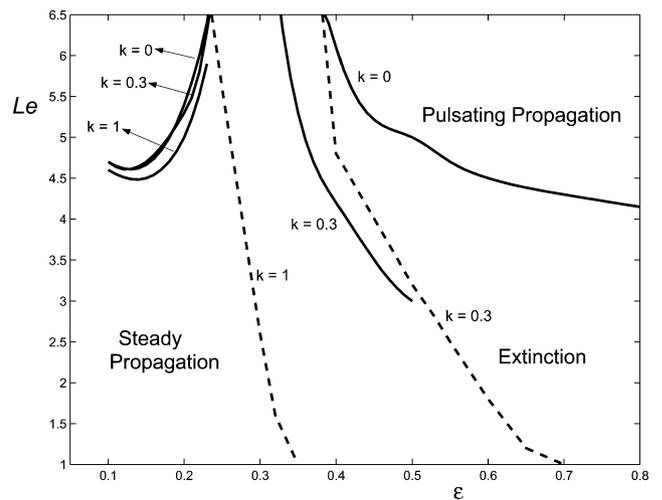
In Fig. 12 we show the variations in the structure of a nonadiabatic pulsating flame at four instances within a cycle. Although the flame front is generally convex toward the unburned gas, there is a noticeable change in curvature during the cycle. As it speeds up, the flame flattens out, and the burning appears more concentrated near the walls. The reverse occurs when the flame slows down: the flame is more curved with the intensity of burning increasing near the center of the channel.

C.   Opposing Flow

We consider now the effect of an imposed flow that opposes the propagation ($u_0 > 0$) on the onset of oscillations. Figure 13 shows the effects of flow velocity and channel width on criticality under adiabatic conditions. The regions below/above the curves correspond, respectively, to steady/pulsating modes of propagation. In channels of moderate width, $0.2 < \epsilon < 0.5$, the critical Lewis number Le_c increases significantly with increasing the flow velocity to values that become unrealistic for common gaseous combustible mixtures. The imposed flow has a relatively small effect on the onset of oscillations in wide channels and an almost negligible effect in narrow channels.

As $\epsilon \to \infty$, the critical value Le_c tends to 3.58, the corresponding value for a one-dimensional planar flame, as it should.

As expected, heat losses lower the critical value of Le_c but also prohibit propagation in narrow channels of width smaller than the corresponding quenching distance. This is illustrated in Fig. 14, calculated for $u_0 = 1$ and selected values of k. As before, the dashed curves are extinction curves, and the solid curves identify the onset of oscillations. For given k, steady propagation is possible below the solid curve and to the left of the corresponding dashed curve. Note that only at near-extinction conditions is Le_c reduced to values that can become accessible in ordinary mixtures.

In Fig. 15 we show the structure of a pulsating curved flame, propagating against an imposed flow with $u_0 = 1$, at four instances during a cycle. The graph on the left corresponds to adiabatic conditions ($k = 0$) and the one on the right to propagation in the presence of heat loss ($k = 1$). Even under adiabatic conditions a dead space develops near the walls as a result of the large Lewis number ($Le = 5$). Heat losses appear to have an insignificant effect in this case because the flame near the walls, where their effect is most important, is anyway quenched. Overall the burning intensity is quite weak: at $t = 1.156$ the propagation speed is at its maximum, and the burning rate varies from ≈ 2.95 at the center of the channel to ≈ 0.05 near the walls; at $t = 1.542$ when the propagation speed is at its minimum, the burning rate varies from ≈ 1.44 at the center to ≈ 0.18 at the
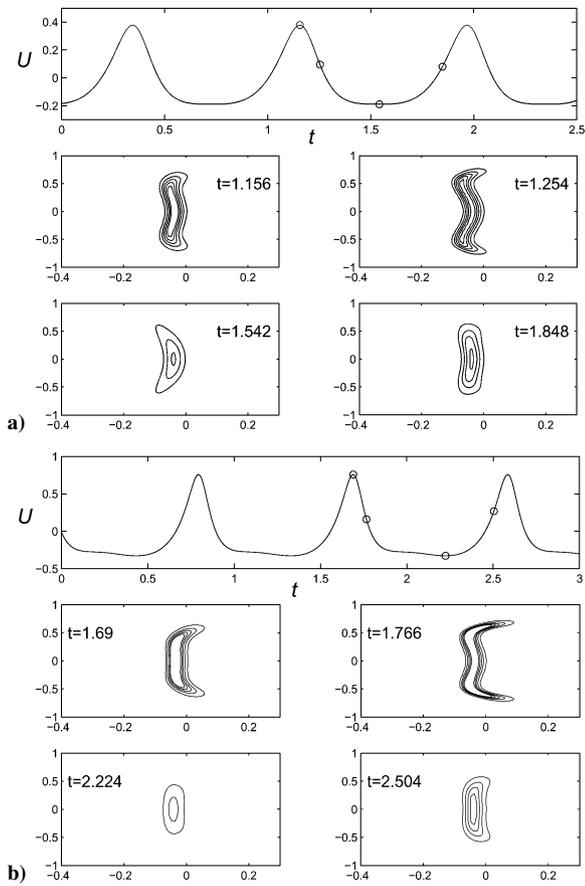
Fig. 15 Reaction-rate contours at four instances [marked in the graph displaying $U(t)$] within a cycle a) for adiabatic conditions and b) for $k = 1$. Calculated for $u_0 = 1$, $\epsilon = 0.1$, and $Le = 5$.
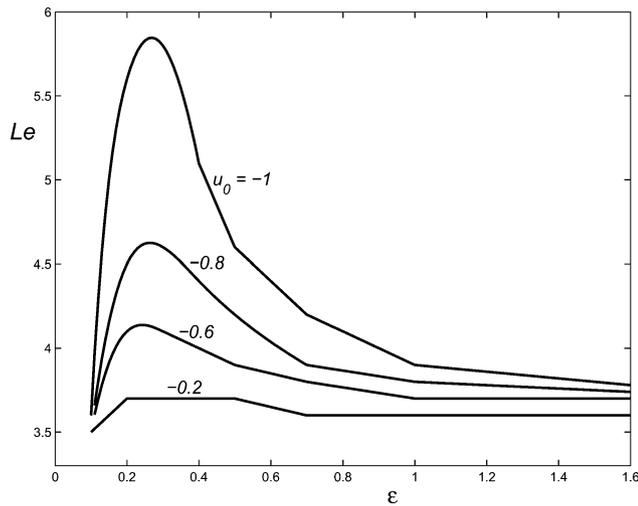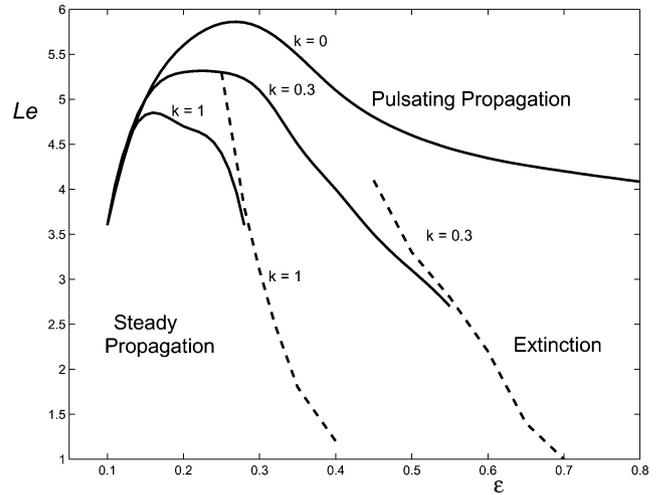

Fig. 16 Regions in the Le vs ϵ parameter plane where steady or pulsating propagation in the presence of a supporting flow under adiabatic conditions occur.

walls. The flame appears localized near the center of the channel in the relatively long time intervals between pulses and comes to life only during the short time intervals with a speed that is close to the adiabatic flame speed. Note that in the case depicted U oscillates between negative and positive values: during the long time intervals of the cycle, the flame is being pushed by the flow, barely surviving, but then flashes back intensely.

D. Supporting Flow

The effects of an imposed flow that supports the propagation ($u_0 < 0$) are discussed next. Figure 16 shows the effects of flow

Fig. 17 Regions in the Le vs ϵ parameter plane where steady or pulsating propagation in the presence of a supported flow under nonadiabatic conditions occur; calculated for $u_0 = -1$.
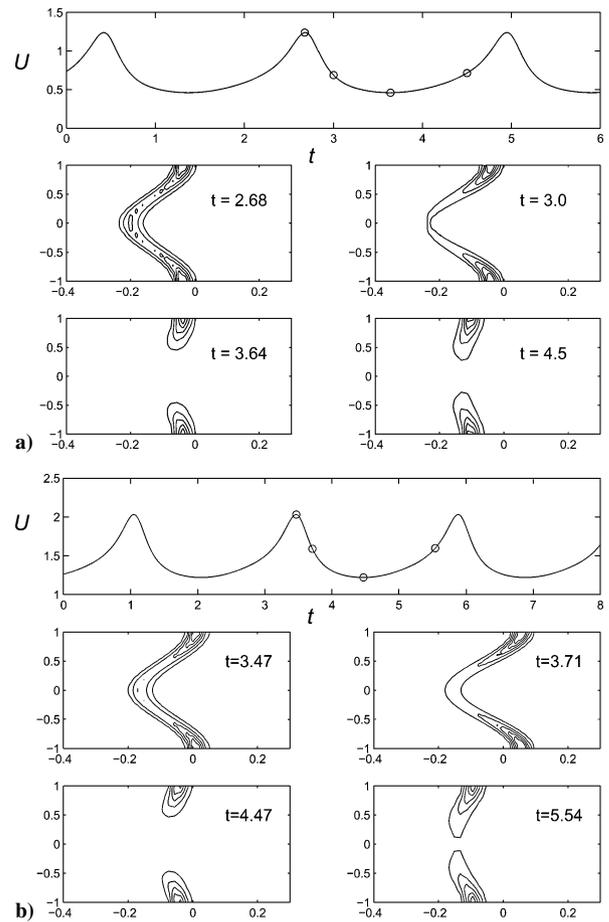
Fig. 18 Reaction-rate contours at four instances [marked in the graph displaying $U(t)$] within a cycle a) for adiabatic conditions and b) for $k = 1$. Calculated for $u_0 = -1$, $\epsilon = 0.1$, and $Le = 4$.

velocity and channel width on criticality under adiabatic conditions. The regions below/above the curves correspond, respectively, to steady/pulsating modes of propagation. Here too the critical Lewis number Le_c increases significantly with increasing the flow velocity to values that become unrealistic for common gaseous combustible mixtures but tends to the value 3.58 for a one-dimensional planar flame, as $\epsilon \to \infty$. The imposed flow has a relatively small effect on the onset of oscillations in wide channels and an almost negligible effect in narrow channels. Regions of steady/pulsating propagation are shown in Fig. 17 for $u_0 = -1$, when heat losses are present. The

dashed curves are the extinction curves, and the solid curves identify the onset of oscillations. Again, we note that the presence of heat losses lowers Le_c to accessible values especially near extinction conditions.

In Fig. 18a we show the structure of a pulsating curved flame, supported by an imposed flow with $u_0 = -1$, at four instances during a cycle for the adiabatic case. The flame spans the entire width of the channel when the speed is at its peak with more intense burning occurring near the walls. During the relaxation period, when the propagation speed is at its lowest value the flame appears nearly extinguished at the center of the channel. For example, when $t = 2.68$ we find the reaction rate varies from ≈ 2.8 near the walls to ≈ 1.5 at the center of the channel; when $t = 3.64$ the burning rate near the walls remains nearly the same ≈ 2.69, but is only ≈ 0.11 at center. Heat losses do not change the flame structure significantly despite the reduction in burning intensity near the walls and the relatively slower average speed, as seen in Fig. 18b.

VI.   Conclusions

Planar flame oscillations have been predicted to occur in premixed combustion systems, when the thermal diffusivity of the mixture is significantly larger than the mass diffusivity of the limiting reactant, namely, when the effective Lewis number is significantly larger than one. A planar flame under such conditions loses its stability and exhibits periodic oscillations with the front moving back and forth normal to itself. Experimental observations of this phenomenon have been primarily verified for burner stabilized flames because the influence of conductive heat loss to the burner's rim acts to lower the critical Lewis number considerably, down to physically realistic values. In this paper we have examined whether heat losses by conduction to the walls of a channel within which the flame is propagating have a similar effect. The problem is more complex, because in the presence of heat losses to the walls the flame is curved and curvature often has a stabilizing influence. Furthermore, as a result of differential diffusion the burning along a two-dimensional flame front is not uniform, and nearly extinguished segments of the front can result. Finally, the onset of oscillations might also depend on the channel's width and on the underlying flowfield.

Our results identify the conditions for the occurrence of a pulsating mode of propagation for two-dimensional flame fronts traveling in a channel. In general, flame oscillations are more likely to be observed in narrow or wide channels; in channels of moderate widths ~ 5–$10 l_{th}$ oscillations are predicted to occur for Lewis numbers excessively large, values that are not physically accessible in common combustible mixtures. Thus, in channels of moderate widths premixed flames are likely to propagate steadily. As expected, heat losses to the walls do promote flame oscillations. With excessive losses, the critical value Le_c is reduced significantly so that oscillations are likely to be observed at near-extinction conditions. When spontaneous oscillations occur at $Le \approx Le_c$, the propagation speed and flame temperature exhibit an oscillatory motion that is nearly sinusoidal in time. For Le significantly larger than Le_c, however, the flame performs relaxation oscillations, characterized by intermittent long and short intervals within a period. During the short intervals, the burning is more vigorous, and the flame speeds up significantly. Within the long intervals the burning intensity is low, and the flame moves very slowly and flattens out.

The presence of convection generally delays the onset of the instability. The mode of oscillation, however, depends on whether the flame is supported or opposed by the flow. The burning is more intense near the center of the channel when the flame is opposed by the flow, with a dead space developing near the walls. When supported by the flow, the burning is more intense near the walls with the flame nearly extinguished at the center of the channel. Local flame extinction results here primarily from differential diffusion effects, with heat losses playing only a separate and often a secondary role. During the relatively long intervals between pulses, the flame appears to be nearly extinguished, but then comes to life during a short time interval moving ahead at a speed comparable or larger than the adiabatic flame speed.

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